Economics 300-01: Quantitative Methods in Economics
Wesleyan University, Fall 2007
Selected Answers to Problem Set #3

4-8: Distribution of likely weather and associated attendance:

Weather	Attendance	Relative Frequency
wet, cold	5,000	0.20
wet, warm	20,000	0.20
dry, cold	30,000	0.10
dry, warm	50,000	0.50

1. The expected (mean) attendance can be calculated by multiplying each likely attendance rate by the probability (i.e. relative frequency) with which it occurs. Thus, if X is the (discrete) random variable representing attendance, then the mean attendance, $\mathrm{E}[X]$ is

$$\begin{aligned} \mathrm{E}[X] &= \sum_{x \in X} x \cdot p(x) \\ &= 5,000(.2) + 20,000(.2) + 30,000(.1) + 50,000(.5) \\ &= 33,000\,. \end{aligned}$$

2. Let's find the expected profit from this venture. The expected revenue is $\$9 \times 33,000 = \$297,000$. The expected (variable) costs are $\$2 \times 33,000 = \$66,000$, and the fixed costs are $\$150,000 + \$60,000 = \$210,000$. The total expected cost is therefore $\$276,000$. Since the expected revenue exceeds the expected cost, the producer should go ahead with the concert — provided he or she is not averse to taking some risk. After all, a profit is not guaranteed by this calculation: it is simply likely.

   (Notice that the break-even point is 30,000 tickets: so long as the producer expects to sell more than this number of tickets, he or she should go ahead with the concert. Do you see how to determine this break-even value?)

4-9: First, let's revise our table to incorporate the updated probabilities, and then recalculate the expected attendance.

Weather	Attendance	Relative Frequency
wet, cold	5,000	0.30
wet, warm	20,000	0.20
dry, cold	30,000	0.20
dry, warm	50,000	0.30

Given these updated probabilities, you can readily confirm that the expected attendance has dropped to $\mathrm{E}[X] = 26,500$ people. How much money will the producer lose if he or she puts on the concert anyway? Let P be the random variable that represents the profit from the concert. Then the expected profit can be seen to be

$$\begin{aligned} \mathrm{E}[P] &= \$9 \cdot \mathrm{E}[X] - \$2 \cdot \mathrm{E}[X] - \$210,000 \\ &= \$238,500 - \$53,000 - \$210,000 \\ &= -\$24,500\,. \end{aligned}$$

On the other hand, if the producer cancels the performance completely, he or she will still have to pay \$30,000 in administrative costs, as well as a \$15,000 penalty. Thus cancelling the concert costs \$45,000.

Since the producer is expected to make a smaller loss from holding the concert than from cancelling it, it makes economic sense for a risk neutral producer to actually hold the concert. (You should calculate the smallest number of tickets the producer could sell that would still make it better — in expectation — to stage the concert than cancel it. If the producer is particularly averse to taking risks, he or she may prefer to simply pay the certain fee of \$45,000 than run the chance of making even bigger losses if the weather turns out to be unfavorable. Notice there is a 30% chance of losing \$175,000, and a 20% chance of losing \$70,000 if the concert is staged. Of course, there is also a 30% chance of turning a \$140,000 profit!)

4-21: We are told that the distribution of price increases (in percentage terms) is approximately normal, with a mean of 8% and a standard deviation of 10%. With X representing the random variable for the percentage increase in housing prices, we can express this distributional assumption symbolically as:

$$X \sim \mathrm{N}(0.08, 0.01)\,.$$

In other words, $\mu = 8\% = 0.08$, and $\sigma = 10\% = 0.1$, so $\sigma^2 = 0.01$.

1. We are asked to find the probability that, given the above distribution, the price of a house will increase by more than 25%. In symbols, we want to find

   $$\Pr(X > 0.25)\,.$$

   In order to solve this problem, we first must standardize the random variable X. Then we will be able to appeal to Table IV in the textbook.

   To standardize a random variable, we subtract its mean and then divide that expression by the standard deviation. (Note: do *not* divide by the variance!) Hence,

   $$\begin{aligned} \Pr(X > 0.25) &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0.25-\mu}{\sigma}\right) \\ &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{0.25-0.08}{0.1}\right) \\ &= \Pr\left(Z > \frac{0.17}{0.1}\right) \\ &= \Pr(Z > 1.7)\,. \end{aligned}$$

   This probability can easily be found in table IV as 0.045: there is a 4.5% chance that they will not be able to afford a house.

2. The chance that the price of a house will drop is the probability that the price increase is less than 0%: $\Pr(X < 0)$. As above, this can be calculated as

   $$\begin{aligned} \Pr(X < 0.0) &= \Pr\left(\frac{X-\mu}{\sigma} < \frac{0.00-0.08}{0.1}\right) \\ &= \Pr(Z < -0.8) \\ &= \Pr(Z > 0.8) \\ &= 0.212\,. \end{aligned}$$

   Therefore, there is a better than 21% chance that they will have “won their gamble handsomely.”

4-22: We are told that the mean is $\mu = 110$ minutes and a standard deviation of $\sigma = 20$ minutes. That is, if X represents the number of minutes it takes to complete the test, $X \sim \mathrm{N}(110, 400)$.

1. The proportion of students who will finish in 120 minutes can be found by determining the percentage of students whose completion time is less than 120 minutes: $\Pr(X < 120)$. Notice

$$\begin{aligned} \Pr(X < 120) &= \Pr\left(\frac{X-\mu}{\sigma} < \frac{120-110}{20}\right) \\ &= \Pr(Z < 0.5) \\ &= 1 - \Pr(Z > 0.5) \\ &= 1 - 0.309 \\ &= 0.691\,, \end{aligned}$$

   or slightly over 69% of the students.

2. To find the time at which exactly 90% of the students will have completed the exam, we must find the value of x_0 such that

$$\Pr(X < x_0) = 0.9\,.$$

   If you draw the distribution, you should realize that this probability statement is equivalent to

$$\Pr(X > x_0) = 0.1\,.$$

   That is, the critical value that gives 90% probability in the left tail is the same critical value that gives 10% probability in the right tail.

   How do we find the value of x_0? First, we need to convert this question into terms of a standard normal distribution:

$$\begin{aligned} \Pr(X > x_0) &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{x_0-\mu}{\sigma}\right) \\ &= \Pr\left(\frac{X-\mu}{\sigma} > \frac{x_0-110}{20}\right) \\ &= \Pr\left(Z > z_0\right)\,, \end{aligned}$$

   where $z_0 = \frac{x_0-110}{20}$.

   For what value of z_0 is $\Pr(Z > z_0) = 0.1$? In table IV, look for the value 0.10 in the blue-shaded area. This corresponds to a value for z_0 of 1.28. Substituting into the above expression, we find $1.28 = \frac{x_0-110}{20}$, or $x_0 = 135.6$. Thus, if the test is roughly two and a quarter hours long, 90% of the students will finish it.

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