Economics 300–01: Quantitative Methods in Economics
Wesleyan University, Fall 2007

Summary of Formulas for Midterm #1

1. **95% Confidence interval for π, the population proportion:**

$$\pi = P \pm 1.96\sqrt{\frac{P(1-P)}{n}},$$

where P is the sample proportion.

2. **Descriptive Statistics:**

   (a) Range = $\max(X_i) - \min(X_i)$

   (b) IQR = 75%ile observation of X_i − 25%ile observation of X_i

   (c) Median = 50%ile observation of X_i

   (d) Sample mean: $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

   (e) Sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n}(X_i)^2 - n\bar{X}^2\right)$

   (f) Sample standard deviation: $s = \sqrt{s^2}$

3. **Probability:**

   (a) Union: $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

   (b) Complement: $\Pr(\bar{A}) = 1 - \Pr(A)$

   (c) Conditional probability (Intersection): $\Pr(A \cap B) = \Pr(A|B)\Pr(B) = \Pr(B|A)\Pr(A)$

   (d) Mutual exclusivity: $\Pr(A \cap B) = 0$

   (e) Independence: $\Pr(A \cap B) = \Pr(A)\Pr(B)$

   (f) Bayes theorem:

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)} = \frac{\Pr(A \cap B)}{\Pr(A \cap B) + \Pr(\bar{A} \cap B)}$$

4. **Univariate Probability Distributions**

   (a) Discrete Random Variables:

   - $\Pr(X = x) = p(x)$, $\quad \sum_x p(x) = 1$
   - Population mean: $\mu = \sum_x x\, p(x) = \mathrm{E}[X]$
   - Population variance: $\sigma^2 = \sum_x (x-\mu)^2\, p(x) = \sum_x x^2\, p(x) - \mu^2 = \mathrm{E}\left[X^2\right] - \mu^2$
   - Population standard deviation: $\sigma = \sqrt{\sigma^2}$
   - $\mathrm{E}\left[g(x)\right] = \sum_x g(x)\, p(x)$

(b) Continuous Random Variables:

- $\Pr(a < X < b) = \int_a^b p(x)\,\mathrm{d}x\,, \quad \int_{-\infty}^{+\infty} p(x)\,\mathrm{d}x = 1$
- Population mean: $\mu = \int_{-\infty}^{+\infty} x\,p(x)\,\mathrm{d}x = \mathrm{E}[X]$
- Population variance: $\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2\,p(x)\,\mathrm{d}x = \mathrm{E}\left[X^2\right] - \mu^2$
- Population standard deviation: $\sigma = \sqrt{\sigma^2}$
- $\mathrm{E}\left[g(x)\right] = \int_{-\infty}^{+\infty} g(x)\,p(x)\,\mathrm{d}x$

(c) Linear Trasformations (discrete or continuous):

- If $Y = a + b\,X$, then
  - $\mathrm{E}[Y] = \mu_Y = a + b\,\mu_X = a + b\,\mathrm{E}[X]$
  - $\sigma_Y^2 = b^2\,\sigma_X^2\,, \quad \sigma_Y = |b|\,\sigma_X$

5. **Binomial Distribution:** $X \sim \mathrm{B}(n, \pi)$

(a) Probability density function:

$$p(x) = \binom{n}{x}\pi^x(1-\pi)^{n-x} = \frac{n!}{x!(n-x)!}\pi^x(1-\pi)^{n-x}\,, \quad x = 0, 1, \ldots, n$$

(b) Mean: $\mu = n\,\pi$

(c) Variance: $\sigma^2 = n\,\pi\,(1-\pi)$

6. **Normal Distribution:** $X \sim \mathrm{N}(\mu, \sigma^2)$

(a) Probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\,, \quad -\infty < x < \infty$$

(b) Mean: $\mathrm{E}[X] = \mu$ (known)

(c) Variance: $\mathrm{E}[X-\mu]^2 = \sigma^2$ (known)

(d) Standard Normal Random Variable:

- $Z = \frac{X-\mu}{\sigma} \sim \mathrm{N}(0,1)$
- $\Pr(a < X < b) = \Pr(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma})$
- $p(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z)^2}\,, \quad -\infty < z < \infty$

7. **Multivariate Probability Distributions** (discrete)

(a) Joint distribution: $\Pr(X = x \cap Y = y) = p(x, y)$

(b) Marginal distributions:

- $\Pr(X = x) = \sum_y p(x, y) = p(x)$
- $\Pr(Y = y) = \sum_x p(x, y) = p(y)$

(c) Independence: $p(x, y) = p(x)\, p(y)$

(d) Conditional Probability:

- $\Pr(Y = y \,|\, X = x) = \frac{p(x,y)}{p(x)} = p(y|x)$
- $\Pr(X = x \,|\, Y = y) = \frac{p(x,y)}{p(y)} = p(x|y)$

(e) Conditional Mean:

- $\mathrm{E}[Y \,|\, X = x] = \sum_y y\, p(y|x)$
- $\mathrm{E}[X \,|\, Y = y] = \sum_x x\, p(x|y)$

(f) $\mathrm{E}[g(X, Y)] = \sum_x \sum_y g(x, y)\, p(x, y)$

(g) Covariance:

$$\begin{aligned}\sigma_{XY} &= \sum_x \sum_y (x - \mu_X)(y - \mu_Y)\, p(x, y) = \mathrm{E}\left[(X - \mu_X)(Y - \mu_Y)\right] \\ &= \sum_x \sum_y x\, y\, p(x, y) - \mu_X \mu_Y = \mathrm{E}[X\, Y] - E[X]\, E[Y]\end{aligned}$$

(h) Correlation: $\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

(i) Linear Trasformations:

- If $W = a + b\, X + c\, Y$, then
  - $\mu_W = a + b\, \mu_X + c\, \mu_Y \quad \Longleftrightarrow \quad \mathrm{E}[W] = a + b\,\mathrm{E}[X] + c\,\mathrm{E}[Y]$
  - $\sigma^2_W = b^2 \sigma^2_X + c^2 \sigma^2_Y + 2\, b\, c\, \sigma_{XY}$

Revised: 26-Sep-2007