**Economics 300–01: Quantitative Methods in Economics**
**Wesleyan University, Spring 2002**

**Midterm Examination #1**
**February 21, 2002**

**Please read the following instructions. Do not begin this examination until instructed to do so.**

1. The total number of points for this examination is 150. The number of points each problem is worth is noted in parentheses. You have 80 minutes to complete this exam.

2. Place all work on the pages of this exam packet. If you need additional space, you may write on the back of a page; clearly indicate which question you are answering.

3. Write all answers clearly. Credit cannot be given for illegible answers. Erase or cross-out mistakes completely.

4. Show your calculations. Partial credit may be awarded where merited. Full credit cannot be awarded for a final answer only. Keep explanations concise.

5. You are allowed to bring the *Summary of Formulas for Midterm #1* to the exam. No other papers may be on your desk. You may use a calculator, but hand-held or portable computers are not permitted.

6. By taking this exam, you agree to comply with Wesleyan University's Honor Code. It is your responsibility to report suspected infractions to this Code. Please sign the pledge below.

   **Pledge: No aid, no violations.**

   Signature: _____

   Print name: _____

   Wes ID number: _____

1. On January 1, 2002, the euro became the sole currency in 12 European nations. Britain has not yet decided to join these other nations in using the euro as its currency. A November 2001 survey of 350 British businesses found that, of those who expressed an opinion, 57% favored joining the European Monetary Union and adopting the euro, while 43% did not. *(16 points)*

   (a) Compute the margin of error for this survey at the 98% level of confidence.

   (b) Assuming the same proportion of responses and a 98% level of confidence, how many firms would have to be interviewed to yield a margin of error of $\pm 3\%$?

2. Consider the following game: For a fee of \$3, you are allowed to toss a fair coin $n$ times. You then get paid $(X^2 - X)$ dollars, where $X$ is the total number of heads tossed. *(20 points)*

   (a) If you are allowed to toss two coins ($n = 2$), what is your expected profit?

   (b) How many coins must you toss to break even in expectation? (That is, for your expected profit to be zero?)

3. Let $A$ and $B$ be two events defined on some sample space. Suppose $\Pr(B|A) > \Pr(B)$. *(20 points)*

   (a) Prove that $\Pr(A) < \Pr(A|B)$.

   (*Hint: If you get stuck, first try to illustrate these results with some actual probabilities. However, credit will be based primarily upon your formal proof.*)

   (b) Prove that $\Pr(A) > \Pr(A|\bar{B})$. (*Hint: $\bar{B}$ is the complement of $B$.*)

4. On a recent (hypothetical) bill before Congress, 400 of the 435 members of the House of Representatives voted as follows: *(22 points)*

| $X$: Party | $Y$: Vote For (0) | $Y$: Vote Against (1) |
|---|---|---|
| Republican (0) | 132 | 88 |
| Democrat (1) | 108 | 72 |

(a) Tabulate the joint and marginal probabilities for this vote.

(b) Compute the correlation between political party and vote.

(c) Is support for this bill independent of political affiliation? Justify your answer.

5. The following quote comes from a February 10, 2002, *New York Times* article entitled "The Painful Fact of Medical Uncertainty": *(20 points)*

> For years, women over 40 years old were told that a yearly mammogram could find breast cancer early enough to save them from death. This was medical dogma; it was the truth. But in the early 1990's, doubts grew about whether the test helped women in their 40's, and now some experts say they question whether it saves anyone....
>
> Some of the strongest evidence is from a study begun in the 1960's. It found that after 18 years, 153 out of 30,131 women who had mammograms had died of breast cancer, and 196 out of 30,565 women who did not have the test died of breast cancer. That is a 30 percent difference in breast cancer death rates — but it hinges on the medical histories of just 43 women. Questions about the design and conduct of this study have led some to doubt its conclusion. And similar questions have been raised about other mammography studies....
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> But many doctors say it is inappropriate for scientists to quibble about the fine points of evidence in the case of a devastating disease like breast cancer. "Unfortunately, the people making these arguments are statisticians," said Dr. Maurie Markman, a specialist in gynecological cancer at the Cleveland Clinic. "I'm not trying to say that statistics are not important," Dr. Markman said. "We can argue about how many angels can dance on the head of a pin. But these are real live patients, and it doesn't help anyone to go through this."

Provide a **brief** reply to Dr. Markman's comments, from a statistical perspective. Keep your comments to *less* than 80 words and *less* than 5 sentences.

6. The noontime air pollution index for downtown Hartford varied as follows for the past 6 months: *(20 points)*

| Index Value | Relative Frequency |
|---|---|
| 0 – 10 | 30% |
| 10 – 20 | 50% |
| 20 – 30 | 8% |
| 30 – 40 | 12% |

(a) Calculate the mean value of this index.

(b) Is the median value of this index above, equal to, or below the mean value computed in part (a)? Justify your answer.

(c) If you instead had six months of daily observations on the air pollution index, would the variance of these daily observations be greater than or less than a variance calculated from the above table? Explain.

7. Let $X$ and $Y$ be independent normally distributed random variables, each with a mean of 2 and a standard deviation of 2. Define the random variable $W = X - Y$. *(32 points)*

   (a) Compute the mean of $W$.

   (b) Compute the variance of $W$.

   (c) A linear function of normally distributed random variables produces a random variable that itself has a normal distribution. Using this fact, the definition of $W$, and your answers to parts (a) and (b) above, compute $\Pr(W \geq 2)$.