Endogenous Persistence and Optimal Monetary Policy Inertia

Michael S. Hanson∗

*Wesleyan University

and

Pavel Kapinos†

Carleton College

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Abstract

Several recent studies have investigated the performance of simple targeting rules, such as nominal income growth targeting or “speed limit” targeting, in ad hoc reduced-form specifications of aggregate supply and aggregate demand that incorporate endogenous persistence. These studies have based their analysis of optimal policy upon a social welfare loss function derived from a model specification that explicitly lacks any persistence. This paper uses a model of endogenous persistence that provides micro-foundations for both the structural dynamic equations and the social welfare loss function. Our simulation results show that, for empirically plausible values of the structural parameters, these simple targeting rules do not necessarily improve upon a pure discretionary policy or optimal inflation targeting. These results contrast with previous studies, such as Jensen (2002) and Walsh (2003).

JEL Categories: E52, E58.

Keywords: Habit formation, inflation persistence, targeting rules, institutional design of monetary policy.

∗ Corresponding author. Department of Economics, 238 Church Street, Middletown, CT 06459. Phone: (860) 685-4634. FAX: (860) 685-2301. E-mail: mshanson@wesleyan.edu

† Department of Economics, Willis Hall 320, Northfield, MN 55057. Phone: (507) 646-7676. FAX: (507) 646-4044. E-mail: pkapinos@carleton.edu

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1 Introduction

A sizable number of central banks around the globe have implemented some form of inflation targeting over the past two decades. Numerous authors have provided theoretical justifications for this policy regime over other potential conceptions of the institutional framework for monetary policy. Recently in this journal, Jensen (2002) and Walsh (2003) have examined the optimality of alternative targeting regimes under discretionary monetary policy.\(^1\) Both of these authors utilize a simple yet common dynamic model of the macroeconomy to suggest that optimal discretionary policy should have an "inertial" form, in the sense that lagged endogenous variables should appear in the loss function to be minimized by the central bank. The basic intuition for these results recognizes that if policy makers could credibly commit themselves to future actions, they could directly influence the formation of inflation expectations today and thereby mitigate the adverse effects of current inflationary shocks in a less costly manner than a prototypical inflation targeting central bank.\(^2\)

Jensen (2002) suggests that a central bank can achieve this goal by targeting the growth rate of nominal income, defined as \(\pi_t + y_t - y_{t-1}\) where \(\pi_t\) is the inflation rate and \(y_t\) is the level of real output, in addition to the gap between actual and potential output, \(x_t \equiv y_t - y^*_{t}\). Targeting nominal income growth links policy actions intertemporally: contractionary policy that lowers real output today will raise the growth rate of nominal (and real) income next period relative to what it would have been. An optimizing discretionary central bank will contract again in this next period (and, inductively, subsequent periods), thereby imparting inertia to the policy action. A similar logic is presented by Walsh (2003), who advocates "speed limit" targeting, in which, in addition to the inflation rate, the central bank targets the growth rate of the output gap, \(x_t - x_{t-1}\), instead of the level, \(x_t\), as in the prototypical inflation targeting regime. Walsh (2003) further investigates how variation in the parameter values of the macroeconomic model influence the desirability of different targeting regimes.\(^3\)

Both of these papers analyze a model economy that can be described by an aggregate supply (e.g. Phillips Curve) relationship for inflation, and an aggregate demand (e.g. IS) relationship for the output

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\(^{1}\) The findings in this literature assume the policy regime is set at the initial period of the analysis, and does not change thereafter. McCallum (1995) offers a critique of this approach; Jensen (2002, p. 931) addresses this issue by appealing to strong constraints on the government’s ability to modify the contract with the central bank that defines the targeting regime.

\(^{2}\) For additional perspectives on this "stabilization bias," see Dennis and Söderström (2002) and McCallum and Nelson (2004). Notice that this source of bias is distinct from the "inflation bias" studied by Barro and Gordon (1983), which is not present in the models examined herein.

\(^{3}\) Thus, both nominal income growth targeting and speed limit targeting (as well as price level targeting) are "flexible" targeting regime in the sense of Svensson (1999).
gap. The canonical New Keynesian (or New Neo-Classical Synthesis) versions of these equations have each variable determined in part by the expected future value of that variable. Following a growing number of other researchers, both Jensen (2002) and Walsh (2003) assume that these two equations exhibit persistence, in the form of lagged values also entering each equation. For moderate, empirically consistent weights on lagged inflation in the AS equation, nominal income growth targeting and speed limit targeting have been shown to outperform inflation targeting in discretionary policy environments, for the reasons outlined above.

More recent research has attempted to provide theoretical foundations for the posited persistence terms in these equations, based on otherwise optimizing behavior by agents. For the Phillips Curve, one common approach has been to assume some degree of indexation by firms who do not get to reset their prices within a Calvo-type price setting framework. Examples of this approach, as well as its implications for optimal monetary policy analysis, can be found in Steinsson (2003) and Rabanal and Rubio-Ramírez (2005). For the IS equation, recent empirical and theoretical work has suggested that habit formation may play an important role in the dynamics of aggregate consumption and output.

In this paper we use a micro-founded approach to incorporate endogenous persistence into a stochastic dynamic macroeconomic model, similar to recent work by Giannoni and Woodford (2003) and Amato and Laubach (2004). Like these authors, we note that the specification of the loss function to be minimized by the central bank can be derived as the second-order approximation to the social welfare function of the representative household. This fact has important implications for investigating optimal monetary policy regimes. First, changes in the utility and production parameters of the micro-foundational framework will not only change the parameters in the log-linearized equations of the model, but also the coefficients in the social loss function. By contrast, Jensen (2002) and Walsh (2003) analyze the impact of variation in the reduced-form parameters of their models for a given parameterization of the social loss function. These authors then separately examine the consequences of changing the relative weights of objectives in the loss function for a given parameterization of their models. A micro-founded approach suggests that one cannot independently vary these components of the system.

Moreover, the second-order approximation of the social welfare function in a model that features endogenous persistence — derived from habit formation and partial price indexation, as discussed above —
will feature lagged values of the output gap and inflation in the central bank loss function, thereby intrinsically incorporating inertial monetary policy. Recall that Jensen (2002) argues in favor of nominal income growth targeting, and Walsh (2003) in favor of speed limit targeting, respectively, because these policies have inertial properties that allow discretionary policy to more closely approximate the optimal (but time inconsistent) precommitment policy in a way that an optimal inflation targeting policy cannot (even with a Rogoff (1985)-style conservative central banker).

The rest of the paper is organized as follows. In section 2 we present a micro-founded model with endogenous persistence in the aggregate demand and aggregate supply relationships. We note there that the ad hoc specifications of Jensen (2002) and Walsh (2003) cannot be derived from micro-foundations. Section 3 discusses the inertial nature of optimal policy in this model. There we contrast the social welfare loss function consistent with the micro-founded model with a simpler, but model-inconsistent, specification used in other studies. We also provide some intuition for the posited desirability of alternative targeting rules that have been proposed elsewhere in the literature. Simulation results that compare these alternative targeting regimes with the optimal discretionary and precommitment policies are reported in section 4. There we show how the desirability of different regimes is highly sensitive to the assumed values of the structural parameters. Section 5 concludes.

2 Model and Calibration

In its most basic form, the analysis of monetary policy regimes requires two pieces: a stochastic model of the economy and a specification of the loss function for society. The central bank is then charged with minimizing the expected present discounted value of societal loss (or, equivalently, maximize the present discounted expected utility), subject to the constraints imposed by the structure of the economy. Both Jensen (2002) and Walsh (2003) model the main equations for the macroeconomy as:

\[ x_t = \theta x_{t-1} + (1 - \theta) E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + u_t, \]  
\[ \pi_t = \phi \pi_{t-1} + (1 - \phi) \beta E_t \pi_{t+1} + \kappa x_t + e_t. \]  

As noted above, \( x_t \) is the output gap, \( (i_t - E_t \pi_{t+1}) \) is the real interest rate, and \( u_t \) is a composite error term that incorporates both traditional "aggregate demand" disturbances (such as exogenous changes in fis-
cal policy) as well as shocks to the (log) level of potential output (such as exogenous changes in technology). \( \theta \) is a parameter that governs the degree of persistence in the output gap; \( \theta = 0 \) corresponds with a forward-looking IS equation that can be derived from the Euler equation for aggregate consumption. Similarly, \( \phi \) measures the degree of persistence in inflation; \( \phi = 0 \) returns the standard New Keynesian Phillips Curve that can be derived, for example, from the Calvo model of price setting. \( \beta \) is the time discount factor, assumed to be the same for private agents and policy makers. \( e_t \) is a “cost-push” shock that generates a trade-off between inflation stabilization and output (gap) stabilization. Both \( u_t \) and \( e_t \) are assumed to follow exogenous stationary AR(1) processes.

The persistence terms in equations (1) and (2) are ad hoc additions, motivated by empirical findings. For example, \( \theta > 0 \) is intended to capture the empirical persistence of output as in, e.g., Rudebusch and Svensson (1999) and Fuhrer (2000); in our model below we explicitly incorporate habit formation in consumption as examined by Fuhrer (2000).\(^6\) Similarly \( \phi > 0 \) is intended to reflect the empirical significance of inflation persistence, as documented by, e.g., Roberts (1997) and Galí and Gertler (1999). As mentioned in the introduction, recent theoretical work has appealed to habit formation to justify the persistence term in equation (1) and partial price indexation to justify the persistence term in equation (2). Below we briefly outline the main equations of a micro-founded model that contains these features. The derivation and solution of the model are very similar to those of Giannoni and Woodford (2003) or Amato and Laubach (2004), and thus are excluded for brevity.\(^7\)

2.1 Microfoundations of Endogenous Persistence

For simplicity, we model a closed economy without capital accumulation. Each representative household owns a monopolistically competitive firm that produces a single differentiated good, which all households in the model economy strictly prefer to consume.\(^8\)

A representative household solves the utility maximization problem:

\[
\max_{\{C_t, N_t, D_{t+1}\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\gamma} (C_t - hC_{t-1})^{1-\gamma} - \frac{N_t^{1+\eta}}{1+\eta} \right\} \right]
\]

\[\text{(3)}\]

\(^6\)Consistent with other macroeconomic approaches to generating persistence in consumption and output, such as Fuhrer (2000), Giannoni and Woodford (2003), and Amato and Laubach (2004), we model “internal” habit formation. Dennis (2004) finds small empirical differences between “internal” and “external” specifications of habit formation.

\(^7\)An appendix with details of the model derivation is available from the authors upon request.

\(^8\)That is, households are assumed to exhibit a preference for variety in consumption. Thus, \( C_t \) can be viewed as a Dixit-Stiglitz composite of the differentiated goods produced by a unit mass of firms.
subject to their intertemporal budget constraint. \(C_t, N_t, \text{ and } D_t\) represent consumption, labor supply, and debt holdings at time \(t\), respectively. Equation (3) includes lagged consumption in the specification of utility: \(h \in [0, 1]\) measures the extent of habit persistence in consumption.

The log-linearized version of the Euler equation that follows from the first-order conditions of the consumer’s maximization problem has the form:

\[
\tilde{c}_t = E_t \tilde{c}_{t+1} - \tilde{y}^{-1}(i_t - E_t \pi_{t+1}),
\]

(4)

where \(\tilde{c}_t\) is defined as:

\[
\tilde{c}_t = (c_t - h c_{t-1}) - \beta h (E_t c_{t+1} - h c_t),
\]

(5)

and \(c_t\) is the log of consumption. \(\tilde{y} = \gamma/(1 - \beta h)\) measures the sensitivity of consumption to the real interest rate; \(\gamma\) is the inverse of the intertemporal elasticity of substitution for consumption. Notice that in the absence of habit formation, \(h = 0\) and equation (4) reduces to the consumption Euler equation commonly found in the literature that lacks endogenous persistence in consumption.

Assuming the government consumes a fixed share of output, subject to mean zero (in logs) stochastic disturbances, equation (4) can be re-written in terms of real output:

\[
\tilde{y}_t = E_t \tilde{y}_{t+1} - \tilde{y}^{-1}(i_t - E_t \pi_{t+1}) + E_t \tilde{g}_{t+1} - \tilde{g}_t.
\]

(6)

The expected change in \(\tilde{g}_t\), the (transformed) innovation to fiscal policy, acts as an aggregate demand shock in this specification. Here, \(\tilde{y}_t\) and \(\tilde{g}_t\) also are defined analogously to \(\tilde{c}_t\) in equation (5).

To rewrite equation (6) in terms of the output gap requires a model of aggregate supply. We assume a simple linear production function, \(y_t = z_t + n_t\) (in log terms), in which the common technological shock, \(z_t\), is assumed to follow an exogenous first-order autoregressive process.\(^{10}\) Equating labor demand and labor supply under flexible prices yields an implicit relationship for the natural level of real output:

\[
\eta y^n_t + \tilde{y}_t y^n_t = (1 + \eta) z_t - \tilde{y} \tilde{g}_t,
\]

(7)

where \(\tilde{y}_t^n\) and \(\tilde{g}_t\) in equation (7) are defined analogously to \(\tilde{c}_t\) in equation (5).

\(^9\)We model the exogenous fiscal process as \(g_t = \rho g_{t-1} + \zeta^g_t\), with \(\zeta^g_t \sim (0, \sigma^2_g)\) and \(0 \leq \rho_g < 1\).

\(^{10}\)That is, analogous to the fiscal shock, the technology shock is modeled as \(z_t = \rho_z z_{t-1} + \zeta^z_t\), with \(\zeta^z_t \sim (0, \sigma^2_z)\) and \(0 \leq \rho_z < 1\).
With equation (7) we can express equation (6) in terms of the (transformed) output gap, \( \tilde{x}_t \equiv \tilde{y}_t - \tilde{y}_n^t \), yielding:

\[
\tilde{x}_t = E_t \tilde{x}_{t+1} - \gamma^{-1}(i_t - E_t \pi_t + r_n^t),
\]

(8)

where \( r_n^t \) is the corresponding natural or “Wicksellian” real rate of interest, defined as:

\[
r_n^t \equiv \gamma \left( E_t \tilde{y}_n^t + 1 - \tilde{y}_n^t + E_t \tilde{g}_t + 1 - \tilde{g}_t \right).
\]

(9)

To facilitate comparison with the ad hoc persistent aggregate demand formulation of equation (1), we can expand the definition of \( \tilde{x}_t \) in equation (8) using the transformation of equation (5) and the law of iterated expectations to give:

\[
x_t = \theta_{-1} x_{t-1} + \theta_{+1} E_t x_{t+1} + \theta_{+2} E_t x_{t+2} - \bar{\sigma} (i_t - E_t \pi_t + r_n^t),
\]

(10)

where \( \theta_{-1} = \left( \frac{h}{1 + h + \beta h^2} \right), \theta_{+1} = \left( \frac{1 + \beta h + \beta h^2}{1 + h + \beta h^2} \right), \theta_{+2} = -\left( \frac{\beta h}{1 + h + \beta h^2} \right), \) and \( \bar{\sigma} = \left( \frac{1 - \beta h}{1 + h + \beta h^2} \right). \)

Importantly, equation (10) is not analogous to equation (1), as the former includes \( E_t x_{t+2} \). Thus, the coefficients on \( x_{t-1} \) and \( E_t x_{t+1} \) on the right-hand side of equation (10) only sum to one in the trivial case in which \( h = 0 \) (i.e. no endogenous persistence). In other words, the version of the aggregate demand relationship that follows from an explicit micro-founded model with habit formation features a qualitatively different form of persistence than the ad hoc specification of equation (1) above. This result can be illustrated quantitatively by noting that the largest possible value for \( \theta_{-1} \), which occurs with \( \beta = 0.99 \) and \( h = 1 \), is 0.3344 — well below the value of \( \theta = 0.5 \) on the lagged output gap in equation (1) assumed by Jensen (2002) and Walsh (2003). Additionally, innovation process, \( r_n^t \), in equations (8) and (10), and the error term, \( u_t \), of equation (1), have different dynamics.

The supply side of the model also delivers a price setting relationship broadly analogous to equation (2). Firms are monopolistically competitive price setters for their differentiated products. We assume Calvo-type nominal price rigidity — standard in the related literature — with \( \alpha \) being the probability that a firm does not adjust its price in a given period. Thus, the average price is fixed for \( 1/(1 - \alpha) \) periods. As in Smets and Wouters (2002) and Rabanal and Rubio-Ramírez (2005), we augment this specification by assuming that a proportion \( \omega \) of firms who do not adjust in a period index their prices to the aggregate price level. This additional assumption yields persistence in the New-Keynesian Phillips Curve
via a lagged inflation term:

\[
\pi_t = \phi_{-1} \pi_{t-1} + \phi_{+1} E_t \pi_{t+1} + \kappa_0 \left( \frac{\eta}{\eta + \gamma} x_t + \frac{\gamma}{\eta + \gamma} \bar{x}_t \right) + \mu_t,
\]

(11)

where \( \phi_{-1} = \frac{\omega}{1 + \omega \beta} \) and \( \phi_{+1} = \frac{\beta}{1 + \omega \beta} \) are the coefficients on lagged and expected future inflation, respectively. Notice that when \( \omega = 0 \) the lagged term disappears from equation (11) and \( \phi_{+1} \) equals \( \beta \), as in the canonical forward-looking New Keynesian Phillips Curve. \( \kappa_0 = \frac{(1-a \beta)[1-a]}{a} \frac{\eta + \gamma}{1 + \omega \beta} \) measures how inflation responds to the output gap when \( h = 0 \). Indeed, for \( h = 0 \) and \( \omega = 0 \), equation (11) reduces to the standard forward-looking New Keynesian Phillips Curve derived from a Calvo price-setting framework. The cost-push shock, \( \mu_t \), can be derived as a stochastic deviation from the steady-state monopolistic markup as in Steinsson (2003). We assume that the cost-push shock also follows an exogenous stationary first-order autoregressive process: \( \mu_t = \rho_{\mu} \mu_{t-1} + \zeta_{\mu} \), with \( \zeta_{\mu} \sim (0, \sigma_{\mu}^2) \) and \( 0 \leq \rho_{\mu} < 1 \).

As is also the case in equation (10), \( h > 0 \) causes lags and expected leads of the output gap to enter into the micro-founded price-setting relationship of equation (11). These terms are missing from the specification of Phillips Curve in Jensen (2002) and Walsh (2003), as represented by equation (2). The only way to eliminate these additional terms is to eliminate endogenous persistence in output; hence the specification of aggregate supply in Jensen (2002) and Walsh (2003), like that of aggregate demand, cannot be reconciled with the micro-foundational model.

### 2.2 Calibration of Parameters

Jensen (2002) and Walsh (2003) largely appeal to empirical macroeconomic evidence to justify their chosen values of the parameters in equations (1) and (2). In this section we investigate evidence on the structural parameters of the above model, noting the (lack of) concordance between the implied reduced form parameters for equations (10) and (11), and those used by Jensen (2002) and Walsh (2003).

Evidence on the degree of habit formation in U.S. data general finds values of \( h \) closer to the upper limit of one: Fuhrer (2000) estimates values between 0.8 and 0.9; Dennis (2004) surveys the literature and finds estimates of \( h \) between 0.54 and 1, while his own estimates on U.S. data are just below 0.9.\(^{11}\)

As noted above, empirically relevant values of \( \beta \) and \( h \) tend to yield estimates of \( \theta_{-1} \) no larger than one-third, well below the assumed value of \( \theta = 0.5 \) in equation (1) of both Jensen (2002) and Walsh (2003).

\(^{11}\)Giannoni and Woodford (2003) assume \( h = 1 \) while Amato and Laubach (2004) adopt Fuhrer’s estimate of \( h = 0.8 \).
Table 1: Baseline Parameterization

| Structural Parameters |  
|----------------------|-------------------|
| \( \beta \)          | 0.99              |
| \( h \)               | 1                 |
| \( \gamma \)          | 1.1               |
| \( \eta \)            | 0.8               |
| \( \alpha \)          | 0.75              |
| \( \omega \)          | 0.5               |
| \( \varepsilon \)     | 8                 |

<table>
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<th>Implied Parameters</th>
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<tr>
<td>( \theta_{-1} )</td>
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</tr>
<tr>
<td>( \theta_{+1} )</td>
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</tr>
<tr>
<td>( \theta_{+2} )</td>
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</tr>
<tr>
<td>( \bar{\sigma} )</td>
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<tr>
<td>( \phi_{-1} )</td>
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</tr>
<tr>
<td>( \phi_{+1} )</td>
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</tr>
<tr>
<td>( \kappa_0 )</td>
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<table>
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<tr>
<th>Exogenous Shock Processes</th>
<th></th>
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<tbody>
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<td>( \rho_g )</td>
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<tr>
<td>( \sigma_g )</td>
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</tr>
<tr>
<td>( \rho_z )</td>
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</tr>
<tr>
<td>( \sigma_z )</td>
<td>0.005</td>
</tr>
<tr>
<td>( \rho_\mu )</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma_\mu )</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Estimates of the elasticity parameters for household utility, \( \gamma \) and \( \eta \), are more diffuse. Fuhrer (2000) estimates \( \gamma \) to be roughly between 6 and 13 for quarterly consumption data in a model does not include labor supply; Dennis (2004) estimates even larger values. On the other hand, Giannoni and Woodford (2003) assume a value of \( \gamma \) of 0.16, based on Rotemberg and Woodford (1997). Part of the difference may be due to lower interest sensitivity of aggregate output than non-durable consumption. For our baseline value of \( \gamma \) we use 1.1 as in Amato and Laubach (2004); in section 4 we explore numerically how our results vary with changes in \( \gamma \). For \( \beta = 0.99 \) and \( h = 1 \), a value for \( \bar{\sigma} \) equal to 1.5 as Walsh (2003) assumes for \( \sigma \) in equation (1) would require \( \gamma = 0.0022 \) — much smaller than any of the above estimates. Notice that \( \bar{\sigma} \) is decreasing in both \( \gamma \) and \( h \): for \( h = 0.5 \), \( \gamma = 0.19265 \) is consistent with \( \bar{\sigma} = 1.5 \).

We set the value of \( \eta \), the Frisch labor supply elasticity, to 0.8 per Dennis (2004), who in turn cites EU estimates from Smets and Wouters (2002). Amato and Laubach (2004) assume \( \eta = 0.6 \). Notice that \( \eta \) affects the simulation in three ways: it mediates the impact of technology and aggregate demand shocks on the natural level of output (equation 9), it influences the slope of the Phillips curve (equation 11), and it affects the relative weight of the output gap in the social loss function (equation 13, below).

On the aggregate supply side, the value of \( \omega \), the fraction of firms that index their prices to the aggregate price level, determines the relative weights given to lagged inflation, \( \phi_{-1} \), and expected future inflation, \( \phi_{+1} \), in equation (11). For \( 0 \leq \omega < \beta \), \( \phi_{-1} \) is strictly less than \( \phi_{+1} \). To match the value of \( \phi = 0.5 \) in Walsh (2003), \( \omega \) must be equal to 0.99, the above-posed value for \( \beta \). While Giannoni and Woodford (2003) assume that \( \omega = 1 \) in their model, estimates of \( \omega \) are generally lower: Rabanal and Rubio-Ramírez
(2005) use Bayesian methods to estimate $\omega = 0.77$, whereas Galí and Gertler (1999) report GMM estimates of $\omega$ between 0.077 and 0.522, depending on the empirical specification. Notice that Jensen's assumed value of $\phi = 0.3$ implies $\omega = 0.4267$ (given $\beta = 0.99$). According to figure 3 of Walsh (2003), the gap between the alternative targeting regimes and an optimal inflation targeting regime is largest for values of $\phi$ between 0.3 and 0.5. Thus, we compromise on $\omega = 0.5$ in our baseline specification, which as shown in table 1 implies $\phi_{-1} = 0.3344$: close to the baseline value assumed by Jensen (2002), and well within the range that Walsh (2003) suggests speed limit and price level targeting dominate inflation targeting.

An extensive literature on Calvo price setting has suggested that $\alpha = 0.75$, which implies prices are fixed on average for $\frac{1}{1-\alpha} = 4$ periods, is a plausible value at a quarterly frequency. Notice that this value of $\alpha$, together with the values for $\beta$, $\omega$, $\gamma$ and $\eta$ discussed above (see table 1), pins down a value for $\kappa_0$ in equation (11). Specifically, $\kappa_0 = 0.1091$ for our baseline parameter values, roughly mid-way between the values for $\kappa$ in equation (2) of 0.05 and 0.142 assumed by Walsh (2003) and Jensen (2002), respectively.

The final structural parameter listed in table 1 is $\varepsilon$, the elasticity of substitution between varieties in the Dixit-Stiglitz aggregator for production. While $\varepsilon$ does not play a direct role in the dynamics of aggregate demand or aggregate supply, it does influence $\tilde{\lambda}$, the relative weight given to the output gap terms in the social loss function of equation (13) below. Our choice of $\varepsilon = 8$ implies an equilibrium markup for the monopolistically competitive firms to be approximately 15%, which coincides with the implied value of Giannoni and Woodford (2003).

The final row of table 1 provides the persistence parameters ($\rho_j$) and the standard deviations ($\sigma_j$) for the three exogenous driving processes of the model. These values are taken directly from Jensen (2002) and Walsh (2003) to facilitate comparisons. Notice that the technology shock is assumed to be very persistent ($\rho_z = 0.97$), whereas the cost-push shock is modeled as white noise ($\rho_\mu = 0$).

### 3 Loss Functions and Policy Regimes

Both Jensen (2002) and Walsh (2003) assume a social loss function of the form:\[12\]

$$L^S = E_t \sum_{t=0}^{\infty} \beta^t \left[ \pi_t^2 + \lambda x_t^2 \right].$$

\[12\] Implicitly, inflation is stabilized around a zero target rate; the socially-optimal level of potential output is equal to the flexible-price level of output, so there is no average inflation bias as in Barro and Gordon (1983); nominal output growth is stabilized around the growth rate of potential output.
Woodford (2003) has shown that this loss function approximates the appropriate social welfare criteria of a New Keynesian model that lacks any endogenous persistence. However, as Giannoni and Woodford (2003) have shown (see also Amato and Laubach, 2004), endogenous persistence in the model leads to lagged values of inflation and/or the output gap appearing in the social loss function: policy is inherently inertial. In particular, one can show that the second-order approximation to the social loss function based on equation (3) has the form:

\[ L = E_t \sum_{t=0}^{\infty} \beta^t \left[ (\pi_t - \omega \pi_{t-1})^2 + \tilde{\lambda} (x_t - \delta x_{t-1})^2 \right], \tag{13} \]

where \( \delta = \frac{h}{\theta} \) measures the contribution of lagged output to the loss function, and \( \tilde{\lambda} = \frac{\theta_0 \gamma (1+\omega \beta)}{\epsilon (\eta + \gamma)} \) is the relative weight on the composite inertial output gap term vis-à-vis a similar inertial term for inflation. Jensen (2002) and Walsh (2003) presume the central bank minimizes equation (12) subject to the dynamic constraints imposed by equations (1) and (2). In contrast with the micro-founded approach presented here, these authors do not establish any link between the specification and parameterization of their loss function and the dynamic equations of their model. As a result, two potential problems arise for their analysis. First, they conduct experiments in which the parameters of the model are varied while the value of \( \lambda \) is held fixed, or vice versa. The micro-foundational approach questions the value of such exercises. Second, their parameterization of the loss function appears to be at odds with that of their model. In their baseline specifications, both impose \( \lambda = 0.25 \) in equation (12) — well below the value for \( \tilde{\lambda} \) implied by the structural parameter values listed in table 1. Furthermore, the only way to justify the exclusion of \( \pi_{t-1} \) or \( x_{t-1} \) from equation (13) and recover the simple form of the loss function of equation (12) is to presume that \( \omega = 0 \) or \( h = 0 \), respectively — i.e., to assume no persistence.

3.1 Optimal Policy: Discretion vs. Precommitment

Optimal monetary policy under precommitment can be described by the following first-order conditions that result from minimizing the objective function in equation (13) subject to the Phillips curve in

\[ \text{footnote 13: That is, the loss function in equation (12) is consistent with equations (1) and (2) only when } \theta = \phi = 0. \]

\[ \text{footnote 14: Amato and Laubach (2003) analyze similar models that feature either rule-of-thumb price setting behavior by firms, or rule-}
\text{of-thumb consumption choices by households — thereby inducing persistence in either the AS or AD equation, respectively — and show that lagged}
\text{values of the variable determined by the rule-of-thumb behavior appear in the social loss function. They do not derive the loss function that}
\text{corresponds with rule-thumb behavior by both types of agents simultaneously.} \]

\[ \text{footnote 15: } \theta = \frac{\beta}{2} \left( \chi + \sqrt{\chi^2 - 4 h^2 \beta^{-1}} \right) \text{is a composite of the structural parameters, with } \chi = \frac{\eta + \gamma (1+\beta h^2)}{\beta \eta}. \text{ This specification of the loss function is conditioned upon the distortions associated with monopolistic competition being arbitrarily close to zero.} \]
equation (11):

\[
2(\pi_t - \omega \pi_{t-1}) - 2 \beta \omega (E_t \pi_{t+1} - \omega \pi_t) - \ell_t + \beta \phi_{t-1} E_t \ell_{t+1} - \phi_{t+1} \beta^{-1} \ell_{t-1} = 0
\]  

(14)

\[
2 \bar{\lambda} (x_t - \delta x_{t-1}) - 2 \beta \bar{\lambda} \delta (E_t x_{t+1} - \delta x_t) + \kappa' [\phi + \gamma] \ell_t - \beta h \gamma E_t \ell_{t+1} - h \gamma \ell_{t-1} = 0
\]  

(15)

where \( \kappa' \equiv \frac{(1-\alpha \beta)(1-\alpha)}{\alpha} = \kappa_0 \left( \frac{1+\omega \beta}{\eta+\gamma} \right) \), and \( \ell_t \) is the Lagrange multiplier associated with the Phillips Curve constraint. Notice that in the absence of endogenous persistence the optimal precommitment policy still would be inertial, as it would still incorporate \( \ell_t \) and \( \ell_{t-1} \). Under discretion, on the other hand, the optimal policy is a simpler “leaning against the wind” rule,

\[
(\pi_t - \omega \pi_{t-1}) = -\frac{\bar{\lambda}}{\kappa'[\phi + \gamma]} (x_t - \delta x_{t-1}),
\]

(16)

which is inertial only because of the presence of lagged values of inflation and the output gap. In the absence of any endogenous persistence (i.e. with \( h = \omega = 0 \)), the optimal discretionary policy solution would reduce to that posited by Clarida et al. (1999), which lacks this inertial nature.

Note that lags and expected leads of variables are featured prominently under precommitment, as is evident from equations (14) and (15). Thus, precommitment policy is inertial for two reasons: First, in contrast with policy under discretion, the central bank can optimize taking into account the agents’ (rational) expectations about future variables — hence the presence of \( \ell_{t-1} \) in equations (14) and (15). Second, the model itself features endogenous persistence. Discretionary policy makers who enact inertial policies, as in equation (16), may be better able to stabilize both inflation and the output gap than a policy based solely on current values, as under the canonical representation of an inflation targeting regime.\(^{16}\) Such is the intuition for the advocacy of nominal income growth targeting by Jensen (2002) and speed limit targeting by Walsh (2003).

### 3.2 Simple Targeting Rules

Based on analyses using a simplified static loss function (of the form in equation 12) combined with their \textit{ad hoc} specification of persistence in equations (1) and (2), both Jensen (2002) and Walsh (2003) demonstrate that alternative targeting regimes that incorporate lagged output terms (either in the form

\(^{16}\)See table 2 for a characterization of inflation targeting. The conditions for optimality for the various discretionary policy regimes we consider are presented in appendix A.
Table 2: Categorization of Targeting Regimes, Walsh (2003)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Loss Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation Targeting</td>
<td>IT $\pi_t^2 + \lambda_{IT} x_t^2$</td>
</tr>
<tr>
<td>Nominal Income Growth Targeting 1</td>
<td>NIT1 $\pi_t^2 + \lambda_{NIT1}(\pi_t + y_t - y_{t-1})^2$</td>
</tr>
<tr>
<td>Nominal Income Growth Targeting 2</td>
<td>NIT2 $\lambda x_t^2 + \lambda_{NIT2}(\pi_t + y_t - y_{t-1})^2$</td>
</tr>
<tr>
<td>Speed Limit Targeting</td>
<td>SLT $\pi_t^2 + \lambda_{SLT}(x_t - x_{t-1})^2$</td>
</tr>
<tr>
<td>Price Level Targeting</td>
<td>PLT $p_t^2 + \lambda_{PLT} x_t^2$</td>
</tr>
</tbody>
</table>

of nominal income growth targeting or speed limit targeting) outperform pure discretionary policy and “optimal” inflation targeting regimes. Table 2 lists the arguments of the central bank’s loss function under different targeting regimes considered by Walsh (2003); the optimal nominal income growth targeting regime proposed by Jensen (2002) generally corresponds with NIT2.\(^\text{17}\)

Rogoff (1985) was among the first in this literature to establish that having the central bank minimize an objective other than the social loss function could improve economic outcomes. In the presence of a Barro-Gordon-type inflationary bias from discretionary policy, a “conservative” central bank — in the sense of $\lambda_{IT} < \lambda$ in equation (12) — could more closely approach the ideal precommitment policy. While discretionary policy in the model considered here does not exhibit average inflation bias, as noted above stabilization bias is a potentially important issue.\(^\text{18}\) Jensen (2002) shows that a “conservative” inflation-targeting central bank can mitigate some of the loss in social welfare due to stabilization bias under discretion.\(^\text{19}\) However, such a policy is trumped by nominal income growth targeting according to Jensen (2002), which in turn is reported to be less desirable than either speed limit targeting or price level targeting — at least for moderate degrees of inflation persistence — by Walsh (2003).

The reason for the superior performance of these alternative targeting rules under discretion is that they induce inertia in policy. For example, in the face of a one-time cost-push shock, a central bank that is targeting nominal income growth will raise interest rates and contract aggregate demand in the current period to offset the shock. With inflation and output returning to their initial values in the next

---

\(^{17}\) The relationship between targeting rules and instrument rules is beyond the scope of this paper. Clarida et al. (1999) and Woodford (2003) show how instrument rules can be derived from targeting rules. McCallum and Nelson (2004) describe how targeting rules can be nested within instrument rules.

\(^{18}\) In a framework similar to that employed by Jensen (2002) and Walsh (2003), McCallum and Nelson (2004) attempt to quantify the welfare costs of the stabilization bias under a purely discretionary policy relative to a “timeless-perspective” policy, and find them to be quantitatively significant for plausible parameter values.

\(^{19}\) A “conservative” central banker is willing to respond to a positive cost-push shock with a deeper recession, thereby stabilizing inflation — and inflation expectations — over the pure discretionary case.
period, expected nominal income growth is temporarily high (since output was depressed by contractionary policy in the period of the cost-push shock) and thus the central bank again contracts. Agents who anticipate this inertial policy response will expect lower future inflation than in the absence of nominal income growth targeting, which improves the policy trade-off and mitigates the stabilization bias. A similar mechanism operates for speed limit targeting.

Since the correct specification of the social loss function for a model with endogenous persistence takes the form of equation (13) instead of (12), the question arises: will the results of Jensen (2002) or Walsh (2003) continue to hold? Moreover, as a pure discretionary policy in this case already incorporates some degree of inertia, it is possible that the simple targeting rules listed in table 2 offer little improvement over the correct specification of optimal discretionary policy. Finally, Walsh (2003) is able to show how the most preferred policy varies with the degree of persistence assumed for inflation, while Jensen (2002) claims that the degree of persistence in the output gap has no bearing on the choice of optimal policy. We explore each of these issues in the simulations that follow.

4 Simulation Results

As noted above, the micro-founded model that we use for our simulations (summarized by equations 10 and 11) does not directly nest the reduced-form specification (equations 1 and 2) of Jensen (2002) or Walsh (2003). Therefore, to facilitate comparisons we conduct two sets of simulations for each parameterization we consider. First, for each candidate policy regime, we compute the value of the social loss function from minimizing equation (13) and report the corresponding standard deviations of inflation ($\pi_t$), the output gap ($x_t$), real output ($y_t$), and the policy instrument, the nominal interest rate ($i_t$). Then we repeat the simulations under the assumption that the central bank minimizes the simple loss function of equation (12). This approach helps to understand the consequences of using a loss function that is not appropriate for the micro-founded model of section 2, and to illustrate the robustness of the analysis to such misspecification of the loss function. Simulations were conducted using the solution technique of Dennis (2003), details of which are available in appendix B.

The central bank minimizes the loss function corresponding to the particular policy regime listed in the second through eighth columns, evaluated at one of the two social loss functions: equation (13) for the odd-numbered tables (“Inertial social loss function”) or equation (12) for the even-numbered tables.
("Simple social loss function"). The first column of results in each table, labeled PC, reports the simulation results under the assumption that the central bank were capable of fully credible precommitment. As in both Jensen (2002) and Walsh (2003), the precommitment result forms the basis for comparison across the various targeting regimes since it does not suffer from stabilization bias. In the presence of cost-push shocks to equation (11), even a credible precommitment policy cannot completely stabilize inflation and the output gap. The second column of results, labeled PD, reports the simulation results for the “pure discretionary” policy; that is, if the central bank treated the social loss function as the description of its policy rule. The remaining columns correspond with the targeting regimes listed in table 2.

The first row in each table below lists the value of the loss function under each policy regime, and the second row lists the loss for each discretionary policy as a percentage of the loss under precommitment. The lower this value, the more effective is the policy regime at minimizing the social welfare loss function. The third row lists the optimal value of the $\lambda_{TR}$ parameter chosen for targeting regime. These values are found via a grid search over the feasible set of values, which in each case is bounded below by zero.

### 4.1 Baseline Parameter Values

Tables 3 and 4 report the simulated results for the baseline parameter values in table 1. In sharp contrast with the results reported in both Jensen (2002) and particularly Walsh (2003), inflation targeting is the best discretionary policy regime of those considered, in that it comes closest to minimizing equation (13) at the precommitment value. The pure discretionary policy is the next best discretionary policy. Speed limit and price level targeting rank third and fourth, respectively, although both involve a premium over the precommitment outcome of more than three times that of the inflation targeting regime. The first nominal income growth targeting regime is not all that much worse than either speed limit targeting or price level targeting, although notice that $\lambda_{NIT1}$ is zero: the optimal regime in this case actually places no weight on nominal income growth; it is thus, in the terminology of Svensson (1999) a “strict” inflation targeting regime. Interestingly, it appears in this case that all three of these targeting regimes are too aggressive in stabilizing inflation, at the cost of greater volatility in the output gap.

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20 “TR” stands for “targeting regime,” and takes on each of the five abbreviations listed in table 2.
21 We set the grid width at 0.01, which matches the degree of precision in Jensen (2002). Experiments with finer grids resulted in no significant change in the reported results but a substantial increase in computational time. Due to the linear-quadratic nature of the problem, a unique minimum exists for each triplet of model parameters, loss function specification, and policy regime that we considered.
22 This interpretation can be verified by noting that the NIT1 regime completely stabilizes inflation: $\text{sd}(\pi_t) = 0$. 

14
**Table 3:** Baseline Parameterization, Inertial social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.3180</td>
<td>0.3412</td>
<td>0.3341</td>
<td>4.0701</td>
<td>0.3783</td>
<td>0.3805</td>
<td></td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>7.30</td>
<td>5.06</td>
<td>21.51</td>
<td>1180</td>
<td>18.96</td>
<td>19.65</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>2.14</td>
<td>0.00</td>
<td>0.01</td>
<td>3.64</td>
<td>5.78</td>
<td></td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.3843</td>
<td>0.5298</td>
<td>0.3715</td>
<td>0.0000</td>
<td>2.2766</td>
<td>0.0816</td>
<td>0.0702</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.4685</td>
<td>0.4557</td>
<td>0.4835</td>
<td>0.5679</td>
<td>0.1188</td>
<td>0.5567</td>
<td>0.5601</td>
</tr>
<tr>
<td>$\text{sd}(y_t)$</td>
<td>3.4069</td>
<td>3.4053</td>
<td>3.4091</td>
<td>3.4221</td>
<td>3.7660</td>
<td>3.4203</td>
<td>3.4207</td>
</tr>
<tr>
<td>$\text{sd}(i_t)$</td>
<td>25.9108</td>
<td>25.9737</td>
<td>25.2414</td>
<td>26.1282</td>
<td>37.0504</td>
<td>24.2524</td>
<td>24.4954</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\omega = 0.5$, $\bar{\lambda} = 1.2793$, and $\delta = 0.9226$.

**Table 4:** Baseline Parameterization, Simple social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.0742</td>
<td>0.0751</td>
<td>0.0747</td>
<td>0.0755</td>
<td>2.8039</td>
<td>0.0752</td>
<td>0.0753</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>1.21</td>
<td>0.67</td>
<td>1.75</td>
<td>3679</td>
<td>1.35</td>
<td>1.48</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>0.15</td>
<td>0.00</td>
<td>0.50</td>
<td>0.79</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.0369</td>
<td>0.0503</td>
<td>0.0304</td>
<td>0.0000</td>
<td>1.6653</td>
<td>0.0184</td>
<td>0.0152</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.5579</td>
<td>0.5569</td>
<td>0.5613</td>
<td>0.5679</td>
<td>0.5460</td>
<td>0.5654</td>
<td>0.5663</td>
</tr>
<tr>
<td>$\text{sd}(y_t)$</td>
<td>3.4203</td>
<td>3.4203</td>
<td>3.4210</td>
<td>3.4221</td>
<td>3.7572</td>
<td>3.4217</td>
<td>3.4217</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\lambda = 0.25$. 
On the other hand, the second nominal income growth targeting regime is far and away the worst performer: it stabilizes the output gap to a greater extent than any of the other policy regimes, but at the cost of substantially more volatility in inflation. Notice from table 2 that inflation enters this targeting regime only as part of the nominal income growth term; since the optimal value of $\lambda_{NIT2}$ is only 0.01, relatively little weight is given to the inflation stabilization objective.

Table 4 reports a similar set of results as table 3, but for the case in which the central bank evaluates its optimal policy with the simple loss function of equation (12) instead of equation (13). Not surprisingly, the reported values of the social loss are very different between tables 3 and 4. Notice that across the regimes, the standard deviation of inflation is uniformly lower — and the standard deviations of the remaining listed variables higher — when the central bank is modeled as minimizing a simple loss function of the form of equation (12) rather than the correctly specified one.

That said, the ranking of regimes by their proximity to the precommitment ideal is no different in table 4 than in table 3, and optimal inflation targeting is still the best discretionary policy regime. However, the optimal $\lambda_{TR}$ weights given to the alternative targeting regimes are much higher in table 4 than table 3, suggesting the use of the simple loss function may over-state the importance of these alternative targeting rules once inertia has been correctly incorporated into the social loss function.

More striking perhaps is the change in the relative performance listed in the second row of table 4: with the simple social loss function, the cost of implementing speed limit targeting or price level targeting appears less than one percentage point greater than the cost of inflation targeting, whereas in table 3 the incremental cost of moving from inflation targeting to either speed limit or price level targeting is nearly 15 percentage points. Thus, using the simple loss function of equation (12) to evaluate alternative targeting regimes may give a very misleading impression of the relative performance across regimes.

One additional noteworthy attribute of tables 3 and 4 is the high volatility of the nominal interest rate relative to the arguments of the loss function (inflation and the output gap). As will be apparent in comparison with subsequent results (see the discussion of tables 11 and 12 below), this result is primarily a consequence of the small value of $\tilde{\sigma}$ implied by the baseline structural parameter values.

### 4.2 Sensitivity to Inflation Persistence

A main finding of Walsh (2003) (also present in Jensen, 2002) is that the preferred policy regime varies with the degree of inflation persistence as measured by the value of $\phi$ in equation (2). In particular,
Table 5: Parameterization with $\omega = 0$, Inertial social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.1423</td>
<td>0.1526</td>
<td>0.1513</td>
<td>0.1729</td>
<td>2.1882</td>
<td>0.1627</td>
<td>0.1647</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>7.24</td>
<td>6.32</td>
<td>21.50</td>
<td>1438</td>
<td>14.34</td>
<td>15.74</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.75</td>
<td>0.00</td>
<td>0.01</td>
<td>7.09</td>
<td>6.29</td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.1478</td>
<td>0.1934</td>
<td>0.1462</td>
<td>0.0000</td>
<td>1.4823</td>
<td>0.0987</td>
<td>0.0894</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.3134</td>
<td>0.3048</td>
<td>0.3246</td>
<td>0.3799</td>
<td>0.0947</td>
<td>0.3579</td>
<td>0.3626</td>
</tr>
<tr>
<td>$\text{sd}(y_t)$</td>
<td>3.3890</td>
<td>3.3884</td>
<td>3.3902</td>
<td>3.3960</td>
<td>3.7626</td>
<td>3.3936</td>
<td>3.3939</td>
</tr>
<tr>
<td>$\text{sd}(i_t)$</td>
<td>17.2961</td>
<td>17.3336</td>
<td>17.5368</td>
<td>17.4900</td>
<td>29.3714</td>
<td>16.2958</td>
<td>16.3819</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\omega = 0$, $\bar{\lambda} = 1.2793$, and $\delta = 0.9226$.

Table 6: Parameterization with $\omega = 0$, Simple social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.0321</td>
<td>0.0333</td>
<td>0.0328</td>
<td>0.0338</td>
<td>2.2275</td>
<td>0.0334</td>
<td>0.0335</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>3.74</td>
<td>2.18</td>
<td>5.30</td>
<td>6839</td>
<td>4.05</td>
<td>4.36</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.14</td>
<td>0.00</td>
<td>0.00</td>
<td>1.35</td>
<td>1.17</td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.0404</td>
<td>0.0555</td>
<td>0.0319</td>
<td>0.0000</td>
<td>1.5000</td>
<td>0.0199</td>
<td>0.0175</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.3611</td>
<td>0.3595</td>
<td>0.3682</td>
<td>0.3799</td>
<td>0.0000</td>
<td>0.3755</td>
<td>0.3765</td>
</tr>
<tr>
<td>$\text{sd}(y_t)$</td>
<td>3.3938</td>
<td>3.3938</td>
<td>3.3947</td>
<td>3.3960</td>
<td>3.3747</td>
<td>3.3955</td>
<td>3.3954</td>
</tr>
<tr>
<td>$\text{sd}(i_t)$</td>
<td>17.5016</td>
<td>17.5075</td>
<td>17.5000</td>
<td>17.4900</td>
<td>17.4900</td>
<td>17.2490</td>
<td>17.2724</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\lambda = 0.25$. 
Walsh (2003) argues that price level targeting performs better than inflation targeting only for values of $\phi < 0.55$ (given his other parameter values), and that speed limit and nominal income growth targeting perform better than inflation targeting only for $\phi < 0.7$. Yet the maximum value of $\phi - 1$ in the micro-founded model is 0.5025 (with $\omega = 1$ and $\beta = 0.99$); thus analyses of the consequences of alternative policy regimes for $\phi$ larger than one half are not relevant to the evaluation of targeting regimes.

We instead investigate the consequences of variation in $\omega$, the fraction of firms who index their prices to the aggregate price level. At one extreme, $\omega = 0$ removes the endogenous persistence from the aggregate supply relationship so that only expected future inflation and the output gap terms appear on the right-hand side of equation (11). Simulations for this case are shown in tables 5 and 6.

Relative to the results in tables 3 and 4, the value of the social loss function at each optimal policy is lower for $\omega = 0$ than for its baseline value of 0.5. This result is not surprising: the greater the extent of inflation persistence, the less influence a given policy action has on the future path of inflation and the more costly is stabilization policy. Concomitantly, the volatility of inflation is lower for each policy regime illustrated in tables 5 and 6 than in the prior results with the baseline parameter values.

Nonetheless, inflation targeting and a pure discretionary policy again are superior to speed limit and price level targeting for minimizing the social welfare loss function, whether modeled correctly as in equation (13) or inconsistently as in equation (12). Also as seen in tables 3 and 4, the absolute difference between inflation targeting and either speed limit or price level targeting is generally much larger for the correctly specified social loss function (table 5) than for the simple one (table 6).

In tables 7 and 8, we move towards the other extreme and examine $\omega = 0.8$, which is at the upper end of the range of empirical estimates of $\omega$ reported in section 2. For the correct (i.e. model consistent) social loss function, table 7 suggests that the pure discretionary policy is preferable to any other discretionary targeting regime. Notice that the pure discretionary policy is the only one that places any weight on lagged inflation: contrast equation (13) with the targeting regime specifications in table 2. For a sufficiently high value of $\omega$, the best discretionary policy is inertial in both inflation and the output gap. Note that even in this case, $\phi - 1$ is only 0.4464 — well within the range that Walsh (2003) reports as favoring the alternative targeting rules. The qualitative results in table 7 otherwise are much the same as in tables 3 and 5; the value of the social loss and the standard deviations of the listed endogenous variables

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18

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23Thus, $\phi + 1 = \beta$. However, equation (11) with $\omega = 0$ does not reduce to equation (2) with $\phi = 0$, as equation (11) still includes lagged and expected future output gap terms.
### Table 7: Parameterization with $\omega = 0.8$, Inertial social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.4569</td>
<td>0.4902</td>
<td>0.4966</td>
<td>0.5552</td>
<td>5.2289</td>
<td>0.5514</td>
<td>0.5526</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>7.29</td>
<td>8.69</td>
<td>21.51</td>
<td>1044</td>
<td>20.68</td>
<td>20.95</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>4.34</td>
<td>0.00</td>
<td>0.02</td>
<td>1.72</td>
<td>5.36</td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.9150</td>
<td>1.3759</td>
<td>0.5492</td>
<td>0.0000</td>
<td>3.2371</td>
<td>0.0486</td>
<td>0.0417</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.5615</td>
<td>0.5462</td>
<td>0.6066</td>
<td>0.6807</td>
<td>0.2578</td>
<td>0.6769</td>
<td>0.6782</td>
</tr>
<tr>
<td>$\text{sd}(y_t)$</td>
<td>3.4209</td>
<td>3.4186</td>
<td>3.4288</td>
<td>3.4426</td>
<td>3.7673</td>
<td>3.4419</td>
<td>3.4420</td>
</tr>
<tr>
<td>$\text{sd}(i_t)$</td>
<td>31.1121</td>
<td>31.2047</td>
<td>29.2848</td>
<td>31.3133</td>
<td>51.8632</td>
<td>29.8631</td>
<td>30.0725</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\omega = 0.8$, $\bar{\lambda} = 1.2793$, and $\delta = 0.9226$.

### Table 8: Parameterization with $\omega = 0.8$, Simple social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>0.1079</td>
<td>0.1082</td>
<td>0.1080</td>
<td>0.1085</td>
<td>2.7993</td>
<td>0.1083</td>
<td>0.1084</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>0.28</td>
<td>0.09</td>
<td>0.56</td>
<td>2494</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.16</td>
<td>0.00</td>
<td>0.24</td>
<td>0.45</td>
<td>1.24</td>
</tr>
<tr>
<td>$\text{sd}(\pi_t)$</td>
<td>0.0258</td>
<td>0.0347</td>
<td>0.0223</td>
<td>0.0000</td>
<td>1.6535</td>
<td>0.0130</td>
<td>0.0100</td>
</tr>
<tr>
<td>$\text{sd}(x_t)$</td>
<td>0.6767</td>
<td>0.6761</td>
<td>0.6778</td>
<td>0.6807</td>
<td>0.6668</td>
<td>0.6797</td>
<td>0.6802</td>
</tr>
<tr>
<td>$\text{sd}(i_t)$</td>
<td>31.1798</td>
<td>31.1898</td>
<td>31.2342</td>
<td>31.3133</td>
<td>74.6652</td>
<td>30.9199</td>
<td>31.0033</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\lambda = 0.25$. 
are greater than in those tables due to the greater inflation persistence.

Importantly, table 8 reveals that with the simple but model-inconsistent social loss function of equation (12), not only does the pure discretionary policy not minimize the simple social loss function (because no weight is given on lagged inflation in that case) but the differences between all the targeting regimes (save for NIT2) appear to be negligible. While the model-consistent social loss function implies a very sizable wedge between the pure discretionary policy and either speed limit or price level targeting (on the order of 14 percentage points), the simple social loss function falsely implies there is nearly no cost of discretionary policy over precommitment — no quantitatively significant stabilization bias — for five of the six discretionary policies considered here.

Based on these results, the purported benefits of alternative policy rules that are inertial only in output (or the output gap) are much lower than figure 3 of Walsh (2003) suggests — indeed, the “benefits” of these policies are negative for our structural parameter values, and become more so as the value of $\omega$ rises. Using the incorrect social loss function for policy evaluation potentially thus could lead to significant misrepresentations of the relative appeal of different targeting rules.

4.3 Sensitivity to Output Gap Persistence

Jensen (2002, p. 932) argues that the value of $\theta$ in equation (1) has no impact upon his results. In the micro-founded model, the degree of persistence in the output gap is determined by the value of $h$. To examine Jensen’s claim, we move from the baseline assumption that $h = 1$ to the other extreme of $h = 0$. This case allows us to examine the model in the absence of any persistence in the aggregate demand specification. Notice that for $h = 0$, equation (10) reduces to a more standard forward-looking IS specification, in which $\tilde{x}_t = x_t$, $\theta_{-1} = \theta_{+2} = 0$, $\theta_{+1} = 1$ and $\tilde{\sigma} = \gamma^{-1}$. Moreover, when $h = 0$, the lagged and expected future output gap terms also disappear from the aggregate supply relationship in equation (11), so that it more closely resembles the simple ad hoc specification of aggregate supply in equation (2).

Tables 9 and 10 report the corresponding simulation results. These are more in the spirit of those reported by Walsh (2003): in table 9 speed limit targeting is the preferred discretionary policy regime, followed by price level targeting and nominal income growth targeting (version 1) with nearly equal values of the social loss function. In contrast with the earlier results, both a pure discretionary policy and inflation targeting perform noticeably worse than most of the alternative targeting rules.\(^\text{24}\) Table 10 indi-

\(^{24}\) Again, NIT2 is a sizable outlier. “$\lambda_{NIT2} > 2000$” signifies that the simulation algorithm hit a boundary in the grid space at $\lambda_{NIT2} =$
Table 9: Parameterization with $h = 0$, Inertial social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>1.6818</td>
<td>2.1602</td>
<td>1.7944</td>
<td>1.7203</td>
<td>2.6858</td>
<td>1.7127</td>
<td>1.7202</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>28.45</td>
<td>6.70</td>
<td>2.29</td>
<td>59.70</td>
<td>1.84</td>
<td>2.28</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.02</td>
<td>0.01</td>
<td>&gt;2000</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>0.8296</td>
<td>1.1236</td>
<td>0.9512</td>
<td>0.7461</td>
<td>1.5627</td>
<td>0.7567</td>
<td>0.8163</td>
</tr>
<tr>
<td>$\sigma(x_t)$</td>
<td>6.4388</td>
<td>7.7842</td>
<td>6.5348</td>
<td>6.9348</td>
<td>2.5962</td>
<td>6.8786</td>
<td>6.4294</td>
</tr>
<tr>
<td>$\sigma(y_t)$</td>
<td>6.7885</td>
<td>8.0758</td>
<td>6.8796</td>
<td>7.2101</td>
<td>3.0506</td>
<td>7.2069</td>
<td>6.7796</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>4.5148</td>
<td>9.0681</td>
<td>5.9166</td>
<td>4.7836</td>
<td>1.2154</td>
<td>4.7859</td>
<td>3.5188</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\omega = 0.5$, $\tilde{\lambda} = 0.0204$, and $\delta = 0$.

Table 10: Parameterization with $h = 0$, Simple social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>30.93</td>
<td>27.33</td>
<td>11.85</td>
<td>5.68</td>
<td>3.73</td>
<td>1.89</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.16</td>
<td>1.85</td>
<td>1.17</td>
<td>0.35</td>
<td>0.78</td>
</tr>
<tr>
<td>$\sigma(\pi_t)$</td>
<td>1.6634</td>
<td>2.0446</td>
<td>1.8813</td>
<td>1.5189</td>
<td>1.6529</td>
<td>1.6910</td>
<td>1.6829</td>
</tr>
<tr>
<td>$\sigma(x_t)$</td>
<td>1.8727</td>
<td>1.5174</td>
<td>2.0830</td>
<td>2.6637</td>
<td>2.1116</td>
<td>1.9088</td>
<td>1.8862</td>
</tr>
<tr>
<td>$\sigma(y_t)$</td>
<td>2.8517</td>
<td>2.6321</td>
<td>2.9940</td>
<td>3.1279</td>
<td>2.7133</td>
<td>2.8755</td>
<td>2.8606</td>
</tr>
<tr>
<td>$\sigma(i_t)$</td>
<td>0.5734</td>
<td>1.8959</td>
<td>2.1937</td>
<td>1.2370</td>
<td>1.2774</td>
<td>0.9167</td>
<td>0.7283</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\lambda = 0.25$. 
icates that price level targeting is the best discretionary targeting policy. However, the use of the model-
inconsistent social loss function in this case does lead to sizable misinterpretations of the relative costs of
inflation targeting and of the two nominal income growth targeting regimes. Additionally, the standard
deviations of inflation are slightly higher, and of all the other variables much lower, than what is actually
socially optimal when the central bank minimizes a social loss function that correctly accounts for the
persistent effects of inflation. Unlike the results in the previous pairs of tables, in this case the value of
the social loss function at each optimal policy is higher in table 10 than in table 9.

With regard to Jensen’s above-cited claim that the degree of output persistence does not affect the
results, a comparison of table 9 with table 3 would seem to suggest otherwise: the ranking of pure dis-
cretion and inflation targeting versus speed limit targeting and price level targeting is reversed, and the
numerical values of the social loss and the standard deviations of the main endogenous variables also
are significantly different across the two tables for each policy regime. There are two primary reasons
for why the effect of changing the degree of persistence here differs significantly from that reported by
Jensen (2002). First, Jensen (2002) holds fixed the value of $\sigma$ while he varies $\theta$. In the micro-founded
model, both are reduced-form parameters that vary with $h$. Second, changes in $h$ affect the aggregate
supply as well as the aggregate demand relationships in the micro-founded model. Again, Jensen (2002)
holds fixed certain reduced-form relationships while varying others in ways that are not consistent with
the micro-foundations of the model in section 2.

The results in tables 9 and 10 more closely resemble those of Jensen (2002) and, especially, Walsh
(2003) in part because with $h = 0$, $\bar{\sigma}$ is approximately 0.91 — much closer to the value of $\sigma = 1.5$ assumed
by Walsh (2003) than is the baseline case. However, since the starting premise of both authors is that real
output does in fact exhibit persistence in the data, these findings represent at best a Pyrrhic victory.

An alternative way to generate a higher value for $\bar{\sigma}$ is to reduce the value of $\gamma$ in the household utility
function. To get $\bar{\sigma}$ in the range of $\sigma$ assumed by Jensen (2002) or Walsh (2003) would require a substan-
tially lower value of $\gamma$ than in our baseline model. In tables 11 and 12 we set $\gamma$ equal to 0.0022; with
$\beta = 0.99$ and $h = 1$, this value for $\gamma$ implies $\bar{\sigma} = 1.5$. The results in these tables are qualitatively similar
to those in tables 9 and 10 above. Price level targeting is the overall best policy, followed by speed limit

\footnotetext{2000. For $1000 \leq \lambda_{NIT2} \leq 2000$, no changes occurred in the third decimal place of either the value of the social loss or the
reported standard deviations. Thus, we are reasonably confident that the reported results are close to the true optimal policy.
Notice that most of the other discretionary policies listed in table 9 have an implicit weight of one on inflation and very low
values for $\lambda_{TB}$. Given the fixed value of $\lambda$ on $x_t$ for NIT2 (see table 2), the optimal policy attempts to place a very large value on
$\lambda_{NIT2}$ to drive down the relative weight on the output gap vis-à-vis inflation. Similar results occur for NIT2 in table 11.

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**Table 11:** Parameterization with $\gamma = 0.0022$, Inertial social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>2.2984</td>
<td>2.7762</td>
<td>2.4861</td>
<td>2.3751</td>
<td>2.5542</td>
<td>2.3643</td>
<td>2.3697</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>20.79</td>
<td>8.17</td>
<td>3.34</td>
<td>11.13</td>
<td>2.87</td>
<td>3.10</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>0.01</td>
<td>0.01</td>
<td>&gt;2000</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>sd($\pi_t$)</td>
<td>1.1273</td>
<td>1.4463</td>
<td>1.1683</td>
<td>1.0267</td>
<td>1.5872</td>
<td>1.0518</td>
<td>1.0903</td>
</tr>
<tr>
<td>sd($i_t$)</td>
<td>3.0263</td>
<td>6.0371</td>
<td>4.8768</td>
<td>2.4540</td>
<td>0.7118</td>
<td>2.4268</td>
<td>1.9952</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\omega = 0.5$, $\tilde{\lambda} = 0.0129$, and $\delta = 0.1854$.

**Table 12:** Parameterization with $\gamma = 0.0022$, Simple social loss function

<table>
<thead>
<tr>
<th>Discretionary Policy Regime</th>
<th>PC</th>
<th>PD</th>
<th>IT</th>
<th>NIT1</th>
<th>NIT2</th>
<th>SLT</th>
<th>PLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Loss</td>
<td>4.8416</td>
<td>5.9596</td>
<td>6.4807</td>
<td>4.6313</td>
<td>5.3210</td>
<td>5.0036</td>
<td>4.9023</td>
</tr>
<tr>
<td>% loss relative to precommitment</td>
<td>—</td>
<td>23.09</td>
<td>33.89</td>
<td>-4.34</td>
<td>9.90</td>
<td>3.35</td>
<td>1.25</td>
</tr>
<tr>
<td>Optimal $\lambda_{TR}$</td>
<td>—</td>
<td>—</td>
<td>1.70</td>
<td>1.56</td>
<td>0.84</td>
<td>0.62</td>
<td>1.52</td>
</tr>
<tr>
<td>sd($\pi_t$)</td>
<td>1.7519</td>
<td>2.4084</td>
<td>1.7712</td>
<td>1.5576</td>
<td>2.0518</td>
<td>2.0826</td>
<td>2.0732</td>
</tr>
<tr>
<td>sd($x_t$)</td>
<td>1.1779</td>
<td>0.9745</td>
<td>3.7299</td>
<td>3.0277</td>
<td>2.1789</td>
<td>1.7158</td>
<td>1.6801</td>
</tr>
<tr>
<td>sd($y_t$)</td>
<td>4.7535</td>
<td>4.6949</td>
<td>5.7613</td>
<td>5.2842</td>
<td>5.0873</td>
<td>4.9027</td>
<td>4.8903</td>
</tr>
<tr>
<td>sd($i_t$)</td>
<td>0.5786</td>
<td>1.5132</td>
<td>1.0847</td>
<td>0.6323</td>
<td>1.1751</td>
<td>0.9829</td>
<td>1.1372</td>
</tr>
</tbody>
</table>

Social loss is multiplied by 100. Standard deviations are in percentages. For this loss function, $\lambda = 0.25$. 

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targeting. However, also as in those cases, the relative sizes of the costs of inflation targeting and of the 
two nominal income growth policies differ sharply between table 11 and table 12, as do the associated 
standard errors of the endogenous variables. Furthermore, the value of the social loss function for the 
optimal nominal income growth targeting regime (NIT1) in table 12 is actually computed to be less than 
that of the precommitment policy when the simple loss function is used. We view this finding as partic-
ularly strong evidence of the inappropriateness of assessing the preferred targeting regime with the loss 
function of equation (12) when the dynamic model of the economy exhibits endogenous persistence in 
both inflation and the output gap.

The lack of robustness for the ranking of optimal targeting rules between tables 9 and 10, and be-
tween tables 11 and 12, is potentially troubling, as it raises the possibility of parameter values for which 
more significant differences occur between the simple, model-inconsistent social loss function and the 
inertial, model-consistent one. In the final three sets of tables (that is, table 7 versus 8, table 9 versus 
10, and table 11 versus 12), the most preferred targeting regime is mis-identified by use of the simple 
(model-inconsistent) social loss function of equation (12) for policy evaluation.

As a final note, the much larger value of $\tilde{\sigma}$ in the simulations of tables 9 through 12 than in the previ-
ous simulations largely explains the sizable differences in the standard deviations of the nominal interest 
rates between these two sets of results. With a higher $\gamma$, consumers have a strong desire to smooth con-
sumption over time, and so an optimizing central bank will aggressively change interest rates to coun-
teract the structural shocks in the model. Similarly, when $h = 1$ consumers exhibit very strong habit 
persistence, and so again volatile interest rates help deflect the impact of the structural shocks on con-
sumption.

5 Conclusion

The evaluation of alternative monetary policy regimes in the face of macroeconomic persistence has 
received significant attention recently. In this paper we have highlighted the importance of the linkages 
between a micro-founded model of endogenous persistence and the institutional design of monetary 
policy. We have emphasized that the specific form of the social welfare loss function can materially affect 
the measurement of the social loss over precommitment of a given discretionary policy rule. Further, we 
have demonstrated that the simple targeting rules proposed in the recent literature do not necessarily
improve on pure discretion or inflation targeting for plausible structural parameter values.

While we concede that our results are derived from one particular — if common — approach to providing micro-foundations, our findings do point to the importance of structural modeling for policy questions. The specification of the model equations has been shown to differ significantly between the micro-founded approach and the more ad hoc reduced-form specifications of prior research, while the coherence of the model — the interrelationship of equations through common structural parameters — is largely absent from these reduced-form approaches. Moreover, the baseline calibration of our micro-founded model, while fairly conventional, is incompatible with the quantitative assumptions of the ad hoc models of, for example, Jensen (2002) and Walsh (2003). These differences have far-reaching implications, especially given that the social welfare loss function is parametrically tied to the structural equations that describe the dynamics of aggregate supply and aggregate demand.

We find evidence that simple targeting rules tend to perform poorly when endogenous persistence has even minimal quantitative significance. Interestingly, this result occurs regardless of whether the social loss function correctly incorporates the true extent of social persistence and regardless of the specific extent of inflation persistence, in contrast to the analysis of Walsh (2003). Contrary to the claims of Jensen (2002), the degree of persistence in output (modeled here as habit formation in consumption) is of paramount importance. The alternative simple targeting rules do outperform inflation targeting and pure discretion in the case where persistence in output, contrary to overwhelming empirical evidence, is negligible. The only case where the simple rules continue to improve welfare in the face of habit formation and inflation persistence is when the coefficient of relative risk aversion is much closer to zero than recent empirical evidence would suggest. The robustness of these simple targeting rules to alternative micro-foundational modeling assumptions is one avenue for future research.
A  Alternative Targeting Regimes and Optimal Policy

In this appendix we briefly discuss the conditions under which the targeting rules listed in table 2 can match the precommitment policy that minimizes the social loss function in equation (12). Thus, the conditions for optimality of these policies are based on the simple model of equations (1) and (2) in the absence of any persistence — that is, with \( \theta = \phi = 0 \). For further details, see Clarida et al. (1999) or Woodford (2003).

Minimizing equation (12) subject to the Phillips Curve in equation (2), with \( \phi = 0 \), yields the following first-order condition that describes the optimal precommitment policy (under a timeless perspective):

\[
\pi_t = -\frac{\lambda}{\kappa} (x_t - x_{t-1}).
\]  

Note that equation (17) is the optimality condition that obtains when the central bank is able to optimize over the agents’ expectations instead of taking them as given as it would under discretion.

Several of the simple targeting rules in table 2 can produce the same specification as in equation (17) while operating under a discretionary procedure. Recall that the above precommitment policy is not time consistent, hence we view the discretionary policy as more relevant for the actual practice of central banks. We consider the policies in table 2 in reverse order, in each case deriving the conditions for optimal policy under discretion.

Under price level targeting, we find:

\[
p_t = -\frac{2\lambda_{PLT}}{\kappa} x_t,
\]

which, when expressed in first differences and with \( \lambda_{PLT} = 0.5\lambda \), should produce the same outcome as in equation (17).

Under speed limit targeting, we find:

\[
\pi_t = -\frac{\lambda_{SLT}}{\kappa} (x_t - x_{t-1}),
\]

which, clearly, for \( \lambda_{SLT} = \lambda \), also should deliver the same outcome as under precommitment in equation (17).
Under the second nominal income growth targeting rule (NIT2), we find:

\[ \pi_t = -(y_t - y_{t-1}) - \frac{\lambda}{\lambda_{NIT2}(1 + \kappa)}(y_t - y^n_t), \]  \hspace{1cm} (18)

where \( y^n_t \) is the natural level of output.\(^{25}\) This rule will be close to equation (17) when \( \lambda \) is small, its ratio to \( \kappa \) is close to 1, and the difference between output growth and output gap growth is small. The latter condition is likely to hold for highly persistent technology shocks that give rise to persistent natural levels of output; recall that \( \rho_z = 0.97 \) in our specification, following Jensen (2002) and Walsh (2003).

Under the first nominal income growth targeting rule (NIT1), we find:

\[ \pi_t = -\frac{\lambda_{NIT1}(1 + \kappa)}{\kappa(1 + \lambda_{NIT1}) + \lambda_{NIT1}}(y_t - y_{t-1}). \]

As above for NIT2, if the difference between the growth rates of real output and of the output gap is relatively small, then NIT1 can bring about results that are very close to equation (17).

As an aside, Jensen (2002) uses a loss function specification that nests NIT1 and NIT2:

\[ \mathcal{L}^J = E_0 \sum_{t=0}^{\infty} \beta^t \{ (1 + f)\pi_t^2 + \lambda x_t^2 + \lambda_J n_t^2 \}. \]

Minimization according to this policy rule yields:

\[ \pi_t = -\frac{(1 + \kappa)\lambda_J}{(1 + f + \lambda_J)\kappa + \lambda_J}(y_t - y_{t-1}) - \frac{\lambda}{(1 + f + \lambda_J)\kappa + \lambda_J}(y_t - y^n_t), \]

which produces the same formulation as equation (18) for \( f = -1 \) and \( \lambda_J = \lambda_{NIT2}. \)

Finally, a “conservative” inflation targeting central banking implies the rule:

\[ \pi_t = -\frac{\lambda_{IT}}{\kappa} x_t, \]

where \( \lambda_{IT} < \lambda. \) Rogoff (1985) has shown when this setup can produce an improvement in social welfare over the case of pure discretion.

\(^{25}\)Note that for nominal income growth targeting, social welfare loss minimization is done with respect to \( y_t \) and not \( x_t. \)
B Solution Algorithm for Simulations

To investigate the nature of optimal policy under both precommitment and discretion, and to better understand how the optimal policy solutions are sensitive to the model specification, we use computational techniques to simulate the model. In particular, we use a version of the technique developed by Dennis (2003) for finding optimal policy in rational expectations models that involve both expectational leads and lags of the endogenous variables. The solution technique proceeds as follows: first, collect the relevant dynamic equations of section 2, as well as any identities necessary to close the model, into the following matrix representation of the structural model:

\[ A_0 y_t = A_1 y_{t-1} + A_2 E_t y_{t+1} + A_3 x_t + A_4 E_t x_{t+1} + A_5 v_t, \]  

where \( y_t \) is the \((n \times 1)\) vector of endogenous variables and \( x_t \) represents the \((p \times 1)\) vector of policy variables. In the simulations reported below, the nominal interest rate, \( i_t \), is assumed to be the sole instrument of policy (i.e., \( p = 1 \)). The three structural shocks in the model — the aggregate demand shock, \( g_t \), the technology shock, \( z_t \), and the cost-push shock, \( \mu_t \) — are included in the \( y_t \) vector and assumed to have exogenous AR(1) representations:

\[
\begin{align*}
g_t &= \rho_g g_{t-1} + \zeta^g_t \\
z_t &= \rho_z z_{t-1} + \zeta^z_t \\
\mu_t &= \rho_\mu \mu_{t-1} + \zeta^\mu_t
\end{align*}
\]

The \((s \times 1)\) matrix \( v_t \) of the independent white-noise forcing shocks \((\zeta^g_t, \zeta^z_t, \text{and } \zeta^\mu_t)\) is distributed as:

\[ v_t \sim i i d(0, \Omega) \]

in which the diagonal elements of \( \Omega \) are \( \sigma^2_{g}, \sigma^2_{z}, \text{and } \sigma^2_{\mu} \), respectively.

The social loss functions of equations (12) and (13) can be expressed in the following general quadratic form:

\[ L = E_0 \sum_{t=0}^{\infty} \beta^t [y_t' W y_t] . \]  

\[ \text{The simulations were computed with code written by the authors for MATLAB version 7 (release 14).} \]

\[ \text{The primary equations of the simulation are (8) and (11), along with the definitions of the flexible-price equilibrium variables in equations (7) and (9). The definitions for the “quasi-differenced” variables, as in equation (5), are included as well.} \]
The central bank then optimally chooses $x_t$ to solve the above linear-quadratic problem, subject to the constraints summarized in equation (19).

While we report both the optimal precommitment and various optimal discretionary policies in the main text, we are most interested in the discretionary solutions. In this case, a stationary solution to the model has the form:

$$y_t = H_1 y_{t-1} + H_2 v_t,$$

$$x_t = F_1 y_{t-1} + F_2 v_t,$$

where equation (21) defines the dynamic updating equation for the variables in the model, and equation (22) represents the (implicit) policy rule as a function of the “state” variables (namely, the exogenous disturbances and the lagged endogenous variables.)

Minimizing the loss function (20) subject to (21) and (22) yields:

$$H_1 = D^{-1}(A_1 + A_3 F_1)$$
$$H_2 = D^{-1}(A_3 + A_5 F_1)$$
$$F_1 = -(A_3' D^{-1} P D^{-1} A_3)^{-1} A_3' D^{-1} P D^{-1} A_1$$
$$F_2 = -(A_3' D^{-1} P D^{-1} A_3)^{-1} A_3' D^{-1} P D^{-1} A_3$$

where

$$D = A_0 - A_2 H_1 - A_4 F_1$$
$$P = W + \beta H_1' P H_1.$$

In this case, Dennis (2003) shows that the loss function (20) under discretion can be computed as:

$$\mathcal{L} = y_t' P y_t + \frac{\beta}{1-\beta} \text{tr} \left[ \Omega H_2' P H_2 \right].$$

The standard errors of the endogenous variables in $y_t$ can be recovered from equation (21), expressed in moving average form.
References


